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Addendum to "Distinguishing Spins in Decay Chains at the Large Hadron Collider"*

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Abstract: We extend our earlier study of spin correlations in the decay chain $D \to Cq$, $C \to Bl^{\text{near}}$, $B \to Al^{\text{far}}$, where A, B, C, D are new particles with known masses but undetermined spins, l^{near} and l^{far} are opposite-sign same-flavour charged leptons and A is invisible. Instead of looking at the observable 2- and 3-particle invariant mass distributions separately, we compare the full three-dimensional phase space distributions for all possible spin assignments of the new particles, and show that this enhances their distinguishability using a quantitative measure known as the Kullback-Leibler distance.

Keywords: Hadronic Colliders, Beyond Standard Model, Supersymmetry Phenomenology, Large Extra Dimensions.

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Contents

1. Introduction

In the recent paper [1], to which we refer the reader for motivation, notation and relevant references, we examined the distinguishability of different spin assignments in the decay chain $D \to Cq$, $C \to Bl^{\rm near}$, $B \to Al^{\rm far}$, where A, B, C, D are new particles with known masses but undetermined spins, $l^{\rm near}$ and $l^{\rm far}$ are opposite-sign same-flavour charged leptons and A is invisible. This was done by comparing separately the invariant mass distributions of the three observable two-body combinations: dileptons (m_{ll}), quark- or antiquark-jet plus positive lepton (m_{jl^+}), and jet plus negative lepton (m_{jl^-}).[1]

If $P(m|S)$ represents the normalized probability distribution of any one of these three invariant masses predicted by spin assignment S, and T is the true spin configuration, then a measure of the improbability of S is provided by the *Kullback-Leibler distance*

$$\mathrm{KL}(T,S) = \int_m \log\left(\frac{P(m|T)}{P(m|S)}\right) P(m|T) dm \; . \tag{1.1}$$

In particular, the number N of events required to disfavour hypothesis S by a factor of $1/R$ under ideal conditions, assuming equal prior probabilities of S and T, would be

$$N \sim \frac{\log R}{\mathrm{KL}(T,S)} \; . \tag{1.2}$$

By ideal conditions we mean isolation of the decay chain with no background and perfect resolution. Therefore N sets a lower limit on the number of events that would be needed in real life. The results for $R = 1000$ are shown in tables 1-3, reproduced for convenience from [1], where a discussion of them can be found. Recall that the notation used is $DCBA$ with F for fermion, S for scalar, V for vector, so that squark decay in SUSY is SFSF and excited quark decay in UED is FVFV. Mass spectra I and II are SUSY- and UED-like respectively (see [1] for details).

[1] The three-body invariant mass m_{jll} was also studied but this is not independent of the two-body masses.

2. Three-dimensional analysis

To extract the most information from the data we should compare the predictions of different spin assigments with the full probability distribution in the three-dimensional space of m_{ll}, m_{jl^+} and m_{jl^-}. The ambiguity between near and far leptons means that this given by

$$
\begin{aligned}
P(m_{ll}, m_{jl^+}, m_{jl^-}) = & \frac{1}{2} f_q \left[P_2(m_{ll}, m_{jl^+}, m_{jl^-}) + P_1(m_{ll}, m_{jl^-}, m_{jl^+}) \right] \\
& + \frac{1}{2} f_{\bar{q}} \left[P_1(m_{ll}, m_{jl^+}, m_{jl^-}) + P_2(m_{ll}, m_{jl^-}, m_{jl^+}) \right] , \qquad (2.1)
\end{aligned}
$$

where f_q and $f_{\bar{q}} = 1 - f_q$ are the fractions of quark- and antiquark-like objects D initiating the decay chain and we use $P_{1,2}(m_{ll}, m_{jl}^{\rm near}, m_{jl}^{\rm far})$ on the right-hand side, assuming both leptons are left-handed, otherwise f_q and $f_{\bar{q}}$ are interchanged. The subscripts 1 and 2 refer to processes 1 and 2 defined in [1] and the factors of one-half enter because $P_{1,2}$ are both normalized to unity.

Instead of trying to evaluate the three-dimensional generalization of the integral in eq. (1.1) analytically, it is convenient to perform a Monte Carlo integration. If we generate m_{ll}, $m_{jl}^{\rm near}$ and $m_{jl}^{\rm far}$ according to phase space, the weight to be assigned to the configuration $l^{\rm near} = l^+$, $l^{\rm far} = l^-$ is

$$
P_{+-}(m_{ll}, m_{jl}^{\rm near}, m_{jl}^{\rm far}) = \frac{1}{2} \left[f_q P_2(m_{ll}, m_{jl}^{\rm near}, m_{jl}^{\rm far}) + f_{\bar{q}} P_1(m_{ll}, m_{jl}^{\rm near}, m_{jl}^{\rm far}) \right] \qquad (2.2)
$$

while that for $l^{\rm near} = l^-$, $l^{\rm far} = l^+$ is

$$
P_{-+}(m_{ll}, m_{jl}^{\rm near}, m_{jl}^{\rm far}) = \frac{1}{2} \left[f_q P_1(m_{ll}, m_{jl}^{\rm near}, m_{jl}^{\rm far}) + f_{\bar{q}} P_2(m_{ll}, m_{jl}^{\rm near}, m_{jl}^{\rm far}) \right] . \qquad (2.3)
$$

In the former case, since the distinction between $l^{\rm near}$ and $l^{\rm far}$ is lost in the data (except when interchanging them gives a point outside phase space), we must use eq. (2.1) with $l^+ = l^{\rm near}$, $l^- = l^{\rm far}$ in the logarithmic factor of the KL-distance, i.e. the contribution is

$$
\log \left(\frac{P_{+-}(m_{ll}, m_{jl}^{\rm near}, m_{jl}^{\rm far}|T) + P_{-+}(m_{ll}, m_{jl}^{\rm far}, m_{jl}^{\rm near}|T)}{P_{+-}(m_{ll}, m_{jl}^{\rm near}, m_{jl}^{\rm far}|S) + P_{-+}(m_{ll}, m_{jl}^{\rm far}, m_{jl}^{\rm near}|S)} \right) P_{+-}(m_{ll}, m_{jl}^{\rm near}, m_{jl}^{\rm far}|T) . \qquad (2.4)
$$

Similarly from the configuration $l^{\rm near} = l^-$, $l^{\rm far} = l^+$ we get the contribution

$$
\log \left(\frac{P_{-+}(m_{ll}, m_{jl}^{\rm near}, m_{jl}^{\rm far}|T) + P_{+-}(m_{ll}, m_{jl}^{\rm far}, m_{jl}^{\rm near}|T)}{P_{-+}(m_{ll}, m_{jl}^{\rm near}, m_{jl}^{\rm far}|S) + P_{+-}(m_{ll}, m_{jl}^{\rm far}, m_{jl}^{\rm near}|S)} \right) P_{-+}(m_{ll}, m_{jl}^{\rm near}, m_{jl}^{\rm far}|T) . \qquad (2.5)
$$

Denoting the sum of these two contributions at the ith phase space point by $\mathrm{KL}_i(T,S)$, and summing over M such points, we have as $M \to \infty$

$$
\frac{M \log R}{\sum_i \mathrm{KL}_i(T,S)} \to N , \qquad (2.6)
$$

which is the Monte Carlo equivalent of eq. (1.2). Results for $R = 1000$ and $M = 5 \times 10^7$ are shown in table 4. By comparing with tables 1-3, we see that, as might be expected, the three-dimensional analysis achieves a discrimination that is better than that of a one-dimensional analysis applied to any single invariant mass distribution. This could be particularly useful in difficult cases like that of distinguishing between SFSF (SUSY) and FVFV (UED).

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References

[1] C. Athanasiou, C. G. Lester, J. M. Smillie and B. R. Webber, arXiv:hep-ph/0605286.

(a)	SFSF	FVFV	FSFS	FVFS	FSFV	SFVF
SFSF	∞	60486	23	148	15608	66
FVFV	60622	∞	22	164	6866	62
FSFS	36	34	∞	16	39	266
FVFS	156	173	11	∞	130	24
FSFV	15600	6864	25	122	∞	76
SFVF	78	73	187	27	90	∞

(b)	SFSF	FVFV	FSFS	FVFS	FSFV	SFVF
SFSF	∞	3353	23	304	427	80
FVFV	3361	∞	27	179	232	113
FSFS	36	44	∞	20	22	208
FVFS	313	184	14	∞	13077	35
FSFV	436	236	15	12957	∞	39
SFVF	89	126	134	38	42	∞

Table 1: The number of events needed to disfavour the column model with respect to the row model by a factor of 0.001, assuming the data to come from the row model, for the $\widehat{m}_{ll}^2$ distribution: (a) mass spectrum I and (b) mass spectrum II.

(a)	SFSF	FVFV	FSFS	FVFS	FSFV	SFVF
SFSF	∞	1059	205	1524	758	727
FVFV	1090	∞	404	3256	4363	1746
FSFS	278	554	∞	418	741	870
FVFS	1605	3242	345	∞	1256	2365
FSFV	749	4207	507	1212	∞	1803
SFVF	813	1821	751	2415	1888	∞

(b)	SFSF	FVFV	FSFS	FVFS	FSFV	SFVF
SFSF	∞	3006	958	6874	761	1280
FVFV	2961	∞	4427	1685	2749	3761
FSFS	914	4201	∞	743	9874	4877
FVFS	6716	1699	752	∞	656	1306
FSFV	720	2666	10279	649	∞	4138
SFVF	1141	3517	5269	1276	4259	∞

Table 2: As in table 1, for the $\widehat{m}_{jl+}^2$ distribution.

(a)	SFSF	FVFV	FSFS	FVFS	FSFV	SFVF
SFSF	∞	1058	505	769	816	619
FVFV	1090	∞	541	5878	4821	445
FSFS	565	714	∞	1032	741	2183
FVFS	799	6435	882	∞	2742	510
FSFV	806	4641	507	2451	∞	413
SFVF	692	541	2272	576	521	∞

(b)	SFSF	FVFV	FSFS	FVFS	FSFV	SFVF
SFSF	∞	3037	689	8633	925	967
FVFV	2985	∞	2271	1431	4368	2527
FSFS	707	2297	∞	526	9874	5004
FVFS	8392	1450	525	∞	653	843
FSFV	924	4287	10279	640	∞	4036
SFVF	1047	2693	5213	870	4041	∞

Table 3: As in table 1, for the $\widehat{m}_{jl-}^2$ distribution.

(a)	SFSF	FVFV	FSFS	FVFS	FSFV	SFVF
SFSF	∞	455	21	47	348	55
FVFV	474	∞	21	54	1387	55
FSFS	33	34	∞	13	39	188
FVFS	55	67	10	∞	54	19
FSFV	341	1339	25	45	∞	66
SFVF	62	64	143	19	79	∞

(b)	SFSF	FVFV	FSFS	FVFS	FSFV	SFVF
SFSF	∞	1053	21	230	194	63
FVFV	1047	∞	27	135	190	90
FSFS	33	42	∞	19	22	175
FVFS	242	140	13	∞	332	33
FSFV	189	194	14	315	∞	37
SFVF	66	95	118	35	41	∞

Table 4: As in table 1, for the combined three-dimensional distribution.